$[Ax + B]' = A$

$[\sqrt{x}]' = \frac{1}{2\sqrt{x}}$

$[c]' = 0$

$[x^a]' = a \cdot x^{-1}$

$[e^x]' = e^x,\ x \in \mathbb{R};$
$[\ln(x)]' = \frac{1}{x},\ x > 0;$
$[\sin(x)]' = \cos(x),\ x \in \mathbb{R};$
$[\mathrm{tg}(x)]' = \frac{1}{\cos^2(x)},\ x \neq \frac{\pi}{2} + k\pi;$
$[\arcsin(x)]' = \frac{1}{\sqrt{1-x^2}},\ x \in (-1, 1);$
$[\mathrm{arctg}(x)]' = \frac{1}{x^2+1},\ x \in \mathbb{R};$
$[\sinh(x)]' = \cosh(x),\ x \in \mathbb{R};$
$[\mathrm{tgh}(x)]' = \frac{1}{\cosh^2(x)},\ x \in \mathbb{R};$
$[\mathrm{argsinh}(x)]' = \frac{1}{\sqrt{x^2+1}},\ x \in \mathbb{R};$
$[\mathrm{argtgh}(x)]' = \frac{1}{1-x^2},\ x \in (-1, 1);$

$[a^x]' = \ln(a)a^x,\ x \in \mathbb{R}.$
$[\log_a(x)]' = \frac{1}{\ln(a)}\frac{1}{x},\ x > 0.$
$[\cos(x)]' = -\sin(x),\ x \in \mathbb{R};$
$[\mathrm{cotg}(x)]' = \frac{-1}{\sin^2(x)},\ x \neq k\pi.$
$[\arccos(x)]' = \frac{-1}{\sqrt{1-x^2}},\ x \in (-1, 1);$
$[\mathrm{arccotg}(x)]' = \frac{-1}{x^2+1},\ x \in \mathbb{R}.$
$[\cosh(x)]' = \sinh(x),\ x \in \mathbb{R};$
$[\mathrm{cotgh}(x)]' = \frac{-1}{\sinh^2(x)},\ x \neq 0.$
$[\mathrm{argcosh}(x)]' = \frac{1}{\sqrt{x^2-1}},\ x \in (1, \infty);$
$[\mathrm{argcotgh}(x)]' = \frac{1}{1-x^2},\ x \in (-\infty, 1) \cup (1, \infty).$

$\int x^a dx = \frac{x^{a+1}}{a+1} + C,\ a \neq -1,$ podmínka dle a.

$\int \frac{1}{x} dx = \ln|x| + C,\ x \neq 0.$

$\int e^x dx = e^x + C,\ x \in \mathbb{R};$

$\int \sin(x) = -\cos(x) + C,\ x \in \mathbb{R};$

$\int \frac{dx}{\cos^2(x)} = \mathrm{tg}(x) + C,\ x \neq \frac{\pi}{2} + k\pi;$

$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + C,\ x \in (-1, 1);$

$\int \sinh(x) = \cosh(x) + C,\ x \in \mathbb{R};$

$\int \frac{dx}{\cosh^2(x)} = \mathrm{tgh}(x) + C,\ x \in \mathbb{R};$

$\int \frac{dx}{\sqrt{x^2+1}} = \mathrm{argsinh}(x) + C,\ x \in \mathbb{R};$

$\int \frac{dx}{x^2-1} = \mathrm{argtgh}(x) + C,\ x \in (-1, 1);$

$\int a^x dx = \frac{a^x}{\ln(a)} + C,\ x \in \mathbb{R}.$

$\int \cos(x) = \sin(x) + C,\ x \in \mathbb{R};$

$\int \frac{dx}{\sin^2(x)} = -\mathrm{cotg}(x) + C,\ x \neq k\pi.$

$\int \frac{dx}{x^2+1} = \mathrm{arctg}(x) + C,\ x \in \mathbb{R}.$

$\int \cosh(x) = \sinh(x) + C,\ x \in \mathbb{R};$

$\int \frac{dx}{\sinh^2(x)} = -\mathrm{cotgh}(x) + C,\ x \neq 0.$

$\int \frac{dx}{\sqrt{x^2-1}} = \mathrm{argcosh}(x) + C,\ x \geq 1;$

$\int \frac{dx}{x^2-1} = \mathrm{argcotgh}(x) + C,\ |x| > 1.$

$e^{\ln(A)} = A$

$a \cdot \ln(A) = \ln(A^a)$

$(A+B)^2 = a^2 + 2ab + b^2$

$(A-B)^2 = a^2 - 2ab + b^2$

$A^2 - B^2 = (a+b)(a-b)$

$(x - x_1)(x - x_2) \quad b^2 - 4ac$

$$\frac{-b \pm \sqrt{D}}{2a}$$

$\ln(A) - \ln(B) = \ln\left(\frac{a}{b}\right)$

$\ln(A) + \ln(B) = \ln(a \cdot b)$

$a \cdot \ln(A) = \ln(A^a)$

ŘADY

podílové kritérium

$$\lambda = \lim_{k \to \infty} \left(\frac{a_k + 1}{a_k}\right) < \begin{matrix} \lambda < 1 \ \Sigma a_k \text{ konverguje} \\ \lambda > 1 \ \Sigma a_k \text{ diverguje } (=\infty) \end{matrix}$$

odmocninové kritérium

$$\rho = \lim_{k \to \infty} \left(\sqrt[k]{a_k}\right) < \begin{matrix} \rho < 1 \ \Sigma a_k \text{ konverguje} \\ \rho > 1 \ \Sigma a_k \text{ diverguje} \end{matrix}$$

srovnávací test

jestliže Σb_k konverguje, pa Σa_k konverguje

jestliže Σa_k ~~konverguje~~ diverguje, pak Σb_k ~~konverguje~~ diverguje

limitní srovnávací test

$a_k \sim b_k \quad \lim_{k \to \infty} \frac{a_k}{b_k} = A > 0$, pak $\Sigma a_k \sim \Sigma b_k$ konverguje

$\infty^L = 0;\ L > 0 \qquad 0^L = 0;\ L > 0$

$L^\infty = \infty;\ L > 1$

$= 0;\ L \langle -1, 1 \rangle$

$=$ neex; $L < -1$

$e^\infty = \infty \qquad \frac{L}{\infty} = 0$

$e^{-\infty} = 0 \qquad \frac{L}{0^\pm} = \pm\infty$

$$T_3 = f_{(a)} + f'_{(a)}(x-a) + \frac{f''_{(a)}(x-a)^2}{2!} + \frac{f'''_{(a)}(x-a)^3}{3!}$$

$2! = 2 \cdot 1$
$3! = 3 \cdot 2 \cdot 1$

- $D_{(f)}$ + spojitost
- lim v krajních bodech $D_{(f)}$ = aby se vědělo, kam to jde
- průsečíky s os. x a y – s os. y se $x = 0$ } dosadit za x 0
  – s os. x se $y = 0$ } pak za y 0
- $f'(x)$ = kritické body
  monotonie = ↗ ↘
  extrémy – lokální max. a min.
- $f''(x)$ = konvexnost / konkávnost = ∩∪
  infelní bod = dosadit krit bod do $f(x)$
- asymptoty – $\lim_{x \to \infty}$ $y = 0$ = vodorovná asympt.
  $y \to \infty$ = neexistuje ⇒ svislá asymp. v x

vyšetřit $\lim_{x \to \infty}$ | $\lim_{x \to -\infty}$ | $\lim_{x \to 0^+}$

TEČNA

$k_T = f'(x)$ v bodě a

$y - f(a) = k_T(x-1)$ ~~tečna~~

NORMÁLA

$k_N = -\frac{1}{k_T}$

$y - f(a) = k_N(x-1)$

fce PROSTÁ: f je prostá, jestliže $\forall x_1, x_2 \, D(f): x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

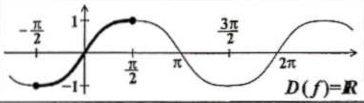
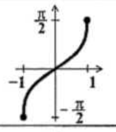
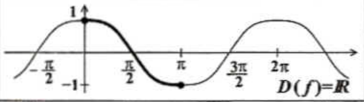
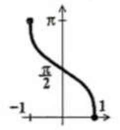
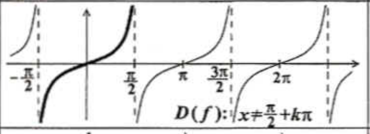
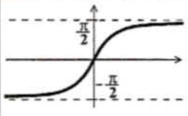
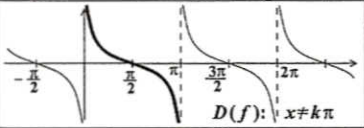
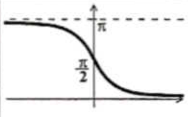
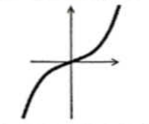
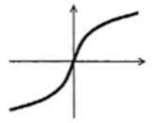
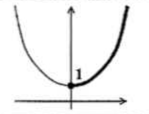
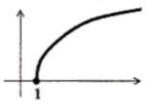
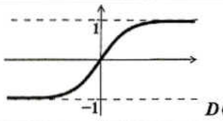
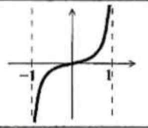
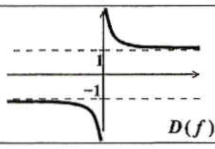
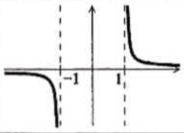
fce INVERZNÍ k f: $g = f_{-1}$, jestliže $\forall x \in D(f): g(f(x)) = x$ a $\forall y \in R(f): f(g(y)) = y$

fce SUDÁ: $D(f)$ musí být symetrický
$\forall x \in D(f): f(-x) = f(x)$

$\cos(-x) = \cos(x)$
$\cos(-\pi) = \cos(\pi) = -1$

fce LICHÁ: $D(f)$ může být symetrický
$\forall x \in D(f): f(-x) = -f(x)$

$\sin(-x) = -\sin(x)$
$(-x)^{17} = -x^{17}$

funkce f	*graf (s prostou restrikcí)*	*vzorce*	f'	*inverze* f_{-1}	*graf*	f_{-1}'
$\sin(x)$ $=\frac{e^{ix}-e^{-ix}}{2i}$ *lichá*, $T=2\pi$	 $D(f)=\mathbb{R}$	$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(2x)=2\sin(x)\cos(x)$ $\quad \sin^2(x)=\frac{1-\cos(2x)}{2}$	$\cos(x)$	$\arcsin(x)$ $D(f_{-1})=\langle -1,1\rangle$		$\frac{1}{\sqrt{1-x^2}}$ $D(f_{-1}')=(-1,1)$
	$\sin(0)=\frac{\sqrt{0}}{2}=0$ $\quad \sin(\frac{\pi}{6})=\frac{\sqrt{1}}{2}=\frac{1}{2}$ $\quad \sin(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$ $\quad \sin(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$ $\quad \sin(\frac{\pi}{2})=\frac{\sqrt{4}}{2}=1$ $\quad \sin^2(x)+\cos^2(x)=1$					
$\cos(x)$ $=\frac{e^{ix}+e^{-ix}}{2}$ *sudá*, $T=2\pi$	 $D(f)=\mathbb{R}$	$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(2x)=\cos^2(x)-\sin^2(x)$ $\quad \cos^2(x)=\frac{1+\cos(2x)}{2}$	$-\sin(x)$	$\arccos(x)$ $D(f_{-1})=\langle -1,1\rangle$		$\frac{-1}{\sqrt{1-x^2}}$ $D(f_{-1}')=(-1,1)$
$\tan(x)$ $=\mathrm{tg}(x)$ $=\frac{\sin(x)}{\cos(x)}$ *lichá*, $T=\pi$	 $D(f)$: $x\neq\frac{\pi}{2}+k\pi$	$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$	$\frac{1}{\cos^2(x)}$	$\arctan(x)$ $=\mathrm{arctg}(x)$ $D(f_{-1})=\mathbb{R}$		$\frac{1}{x^2+1}$ $D(f_{-1}')=\mathbb{R}$
$\mathrm{cotg}(x)$ $=\frac{\cos(x)}{\sin(x)}$ *lichá*, $T=\pi$	 $D(f)$: $x\neq k\pi$	$\mathrm{cotg}(x+y)=\frac{\mathrm{cotg}(x)\mathrm{cotg}(y)-1}{\mathrm{cotg}(x)+\mathrm{cotg}(y)}$ $\mathrm{cotg}(2x)=\frac{\mathrm{cotg}^2(x)-1}{2\,\mathrm{cotg}(x)}$	$\frac{-1}{\sin^2(x)}$	$\mathrm{arccotg}(x)$ $D(f_{-1})=\mathbb{R}$		$\frac{-1}{x^2+1}$ $D(f_{-1}')=\mathbb{R}$
$\sinh(x)$ $=\frac{e^x-e^{-x}}{2}$ *lichá*	 $D(f)=\mathbb{R}$	$\sinh(x+y)=\sinh(x)\cosh(y)+\cosh(x)\sinh(y)$ $\sinh(2x)=2\sinh(x)\cosh(x)$ $\sinh^2(x)=\frac{\cosh(2x)-1}{2}$	$\cosh(x)$	$\mathrm{argsinh}(x)$ $=\ln\left(x+\sqrt{x^2+1}\right)$ $D(f_{-1})=\mathbb{R}$		$\frac{1}{\sqrt{x^2+1}}$ $D(f_{-1}')=\mathbb{R}$
		$\cosh^2(x)-\sinh^2(x)=1$				
$\cosh(x)$ $=\frac{e^x+e^{-x}}{2}$ *sudá*	 $D(f)=\mathbb{R}$	$\cosh(x+y)=\cosh(x)\cosh(y)+\sinh(x)\sinh(y)$ $\cosh(2x)=\cosh^2(x)+\sinh^2(x)$ $\cosh^2(x)=\frac{\cosh(2x)+1}{2}$	$\sinh(x)$	$\mathrm{argcosh}(x)$ $=\ln\left(x+\sqrt{x^2-1}\right)$ $D(f_{-1})=\langle 1,\infty)$		$\frac{1}{\sqrt{x^2-1}}$ $D(f_{-1}')=(1,\infty)$
$\tanh(x)$ $=\mathrm{tgh}(x)$ $=\frac{\sinh(x)}{\cosh(x)}$ *lichá*	 $D(f)=\mathbb{R}$	$\tanh(x+y)=\frac{\tanh(x)+\tanh(y)}{1+\tanh(x)\tanh(y)}$ $\tanh(2x)=\frac{2\tanh(x)}{1+\tanh^2(x)}$	$\frac{1}{\cosh^2(x)}$	$\mathrm{argtanh}(x)$ $=\mathrm{argtgh}(x)$ $=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ $D(f_{-1})=(-1,1)$		$\frac{1}{1-x^2}$ $D(f_{-1}')=(-1,1)$
$\mathrm{cotgh}(x)$ $=\frac{\cosh(x)}{\sinh(x)}$ *lichá*	 $D(f)$: $x\neq 0$	$\mathrm{cotgh}(x+y)=\frac{1+\mathrm{cotgh}(x)\mathrm{cotgh}(y)}{\mathrm{cotgh}(x)+\mathrm{cotgh}(y)}$ $\mathrm{cotgh}(2x)=\frac{1+\mathrm{cotgh}^2(x)}{2\mathrm{cotgh}(x)}$	$\frac{-1}{\sinh^2(x)}$	$\mathrm{argcotgh}(x)$ $=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ $D(f_{-1})$: $\lvert x\rvert>1$		$\frac{1}{1-x^2}$ $D(f_{-1}')$: $\lvert x\rvert>1$